

# A Simple and Realistic Model of Supersymmetry Breaking

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## Abstract

We present a simple and realistic model of supersymmetry breaking. In addition to the minimal supersymmetric standard model, we only introduce a hidden sector gauge group  $SU(5)$  and three fields  $X$ ,  $F$  and  $\bar{F}$ . Supersymmetry is broken at a local minimum of the potential, and its effects are transmitted to the supersymmetric standard model sector through both standard model gauge loops and local operators suppressed by the cutoff scale, which is taken to be the unification scale. The form of the local operators is controlled by a  $U(1)$  symmetry. The generated supersymmetry breaking and  $\mu$  parameters are comparable in size, and no flavor or  $CP$  violating terms arise. The spectrum of the first two generation superparticles is that of minimal gauge mediation with the number of messengers  $N_{\text{mess}} = 5$  and the messenger scale  $10^{11} \text{ GeV} \lesssim M_{\text{mess}} \lesssim 10^{13} \text{ GeV}$ . The spectrum of the Higgs bosons and third generation superparticles, however, can deviate from it. The lightest supersymmetric particle is the gravitino with a mass of order  $(1 - 10) \text{ GeV}$ .

# 1 Introduction

Weak scale supersymmetry has long been the leading candidate for physics beyond the standard model. It not only stabilizes the Higgs potential against potentially large radiative corrections, but also provides a successful prediction for the weak mixing angle through gauge coupling unification [1]. This framework, however, also introduces several new mysteries. Chief amongst these are:

- What is the origin of supersymmetry breaking, whose scale is hierarchically smaller than the Planck scale?
- Why is the supersymmetric mass for the Higgs doublets ( $\mu$  parameter) the same order of magnitude as the supersymmetry breaking masses?
- Why have we not already observed flavor changing or  $CP$  violating processes that are expected to occur in generic weak scale supersymmetric theories?
- Why does the proton not decay very rapidly through the processes that are allowed in general supersymmetric theories?

In this paper we present a simple and realistic model of weak scale supersymmetry which addresses these questions.

The model we consider is very simple. In addition to the minimal supersymmetric standard model (MSSM), we only introduce a hidden sector gauge group  $G_{\text{hid}} = SU(5)_{\text{hid}}$  and three fields  $X$ ,  $F$  and  $\bar{F}$ . The quantum numbers of these fields under  $SU(5)_{\text{hid}} \times G_{\text{SM}}$ , where  $G_{\text{SM}}$  is the standard model gauge group, are  $X(\mathbf{1}, \mathbf{1})$ ,  $F(\mathbf{5}, \mathbf{5}^*)$  and  $\bar{F}(\mathbf{5}^*, \mathbf{5})$ . Here, we have used the language of  $SU(5)_{\text{SM}} \supset G_{\text{SM}}$  for simplicity, although  $G_{\text{SM}}$  does not have to be unified into a single gauge group. The model has the superpotential interaction

$$W = \lambda X F \bar{F}, \quad (1)$$

as well as the Kähler potential interactions  $K = -|X|^4/4M_*^2 + (X^\dagger H_u H_d/M_* + \text{h.c.})$ , where  $H_u$  and  $H_d$  are the two Higgs doublets of the MSSM, and  $O(1)$  coefficients are omitted. The scale  $M_*$  suppressing the Kähler potential interactions (the effective cutoff scale) is taken to be the unification scale,  $M_* \simeq 10^{16}$  GeV. We find that this simple structure, together with a  $U(1)$  symmetry controlling the form of the interactions, is essentially all we need to address the questions listed above.

The scale of supersymmetry breaking in our model is generated dynamically [2]. With the interaction of Eq. (1), the dynamics of  $G_{\text{hid}}$  generates the effective superpotential  $W_{\text{eff}} = \lambda X \Lambda^2$  for  $\lambda X \gtrsim \Lambda$ , where  $\Lambda$  is the dynamical scale of  $G_{\text{hid}}$  [3, 4]. This breaks supersymmetry on a plateau of the potential at  $X \gtrsim \Lambda/\lambda$ , with the scale of supersymmetry breaking given by

$|F_X|^{1/2} = \lambda^{1/2}\Lambda$ . The field  $X$  is a pseudo-flat direction, and we consider stabilizing it using supergravity effects. In particular, we consider the  $X$  mass term arising from the higher order Kähler potential term,  $-|X|^4/4M_*^2$ , and the  $X$  linear term appearing in supergravity [5, 6]. This gives  $\langle X \rangle \approx M_*^2/M_{\text{Pl}}$ , where  $M_{\text{Pl}} \simeq 10^{18}$  GeV is the reduced Planck scale. With this value of  $\langle X \rangle$ , the  $\mu$  parameter arising from the Kähler potential [7] and the supersymmetry breaking masses arising from integrating out the  $F, \bar{F}$  fields [8, 9] are comparable [10]. No flavor violating or  $CP$  violating effects arise. The particular form of the superpotential and the Kähler potential of the model is enforced by a global  $U(1)$  symmetry under which  $X, F, \bar{F}, H_u$  and  $H_d$  carry the charges of 2,  $-1$ ,  $-1$ , 1 and 1, respectively. (The charges of the matter fields are chosen accordingly.) This symmetry is crucial to control the size of the  $\mu$  parameter. The same  $U(1)$  symmetry can also be used to forbid dangerous operators leading to rapid proton decay.

The model provides a complete description of the dynamics, including supersymmetry breaking and its mediation, below the effective cutoff scale of  $M_* \simeq 10^{16}$  GeV. We find that the coupling  $\lambda$  in Eq. (1) must be in the range  $10^{-6} \lesssim \lambda^2 \lesssim 10^{-3}$ . This and other properties of the model lead to several testable predictions. In particular,

- The spectrum of the first two generation squarks and sleptons is that of minimal gauge mediation with the number of messengers  $N_{\text{mess}} = 5$  and the messenger scale  $10^{11}$  GeV  $\lesssim M_{\text{mess}} \lesssim 10^{13}$  GeV.<sup>1</sup>
- The Higgs soft masses,  $m_{H_u}^2$  and  $m_{H_d}^2$ , and the third generation squark and slepton masses deviate from those of minimal gauge mediation, but the deviations are parameterized by two free parameters.
- The lightest supersymmetric particle is the gravitino with a mass of order  $(1 - 10)$  GeV.

We also find that the model presented here can be naturally “derived” by making a series of simple hypotheses for solutions to the issues listed at the beginning. We consider that the existence of such an argument, together with the simplicity of its structure, makes the model very attractive.

In the next section, we provide this argument. The actual model is presented and analyzed in the following three sections: sections 3, 4 and 5. The  $U(1)$  symmetry of the model is elaborated further in section 6. Finally, a summary and discussions are given in section 7, where the robustness of the predictions under possible modifications of the model is discussed.

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<sup>1</sup>After submitting this paper, Ref. [20] appeared which claims that the messenger scale must be smaller than about  $10^{10}$  GeV. We disagree with this. By choosing  $M_* \simeq 2 \times 10^{16}$  GeV and  $\lambda \simeq 5 \times 10^{-3}$ , for example, we can obtain a realistic phenomenology with  $M_{\text{mess}} \simeq 10^{12}$  GeV, as can be seen from the analysis in sections 3 and 4. The paper [20] also studied the range  $M_{\text{mess}} \lesssim 10^{10}$  GeV, which is outside the regime of validity of our present analysis, and found a consistent supersymmetry breaking minimum in a branch of the moduli space. Overall, the range of the messenger scale in the present model is, then,  $10^5$  GeV  $\lesssim M_{\text{mess}} \lesssim 10^{13}$  GeV (except possibly for values around  $M_{\text{mess}} \approx (10^{10} - 10^{11})$  GeV, where there is no theoretical control over the dynamics).

## 2 “Derivation”

One of the most promising ways to address why the  $\mu$  parameter is the same order as the supersymmetry breaking masses is to forbid  $\mu$  in the supersymmetric limit and generate it through supersymmetry breaking. There are two classes of symmetries that can achieve this. One is an  $R$  symmetry under which the Higgs bilinear  $H_u H_d$  is neutral. The other is a non- $R$  symmetry under which  $H_u H_d$  is charged. (An  $R$  symmetry under which  $H_u H_d$  carries a nonzero charge other than 2 also falls in this latter class.) In these cases,  $\mu$  can be generated by coupling  $H_u H_d$  to the supersymmetry breaking superfield  $X = \theta^2 F_X$  in the Kähler potential

$$K = \frac{1}{M_*} X^\dagger H_u H_d + \text{h.c.}, \quad (2)$$

where  $M_*$  is some mass scale [7]. The  $X$  field is neutral in the former case, while it has the same nonzero charge as  $H_u H_d$  in the latter case. An interesting point is that in either case the symmetry that forbids a (potentially large) Higgs mass in the supersymmetric limit also forbids a linear  $X$  term in the superpotential,  $W = M^2 X$ , which contributes to supersymmetry breaking with the (potentially large) breaking scale,  $M$ . The reason why the Higgs doublet mass is not the Planck scale is related to the reason why supersymmetry is not broken at the Planck scale.

Let us now adopt the latter case: a non- $R$   $U(1)$  global symmetry,  $U(1)_H$ , forbidding the  $H_u H_d$  and  $X$  terms in the superpotential. This has an advantage that the Kähler potential term  $K = X^\dagger X H_u H_d / M_*^2 + \text{h.c.}$  is not allowed, so that the holomorphic supersymmetry breaking Higgs mass-squared ( $B\mu$  parameter) is not generated at the same order as  $\mu^2$ . This avoids generating problematic large  $CP$  violation at low energies, such as an electron electric dipole moment beyond the current experimental limit, which would arise if both  $\mu$  and  $B\mu$  were generated with a comparable order and arbitrary complex phases.<sup>2</sup> The question then is how to generate a linear  $X$  term in the superpotential, needed to break supersymmetry, and a coupling of  $X$  to the standard model gauge supermultiplets, needed to generate the gaugino masses in the MSSM. These operators are both forbidden by the  $U(1)_H$  symmetry.

It is immediately clear that if  $U(1)_H$  has anomalies with respect to the hidden sector gauge group  $G_{\text{hid}}$  and the standard model gauge group  $G_{\text{SM}}$ , the low energy effective theory has the operators

$$\mathcal{L} = \int d^2\theta \frac{A_{\text{hid}}}{32\pi^2} (\ln X) \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.}, \quad (3)$$

and

$$\mathcal{L} = \sum_a \int d^2\theta \frac{A_{\text{SM}}}{32\pi^2} (\ln X) \mathcal{W}^{a\alpha} \mathcal{W}_\alpha^a + \text{h.c.}, \quad (4)$$

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<sup>2</sup>For  $M_* \approx M_{\text{Pl}}$ ,  $B\mu \approx F_X^2 / M_* M_{\text{Pl}}$  generated by gravity mediation can be comparable to  $\mu^2 \approx F_X^2 / M_*^2$ . This contribution, however, does not necessarily introduce a new  $CP$  violating phase. Moreover, the generated  $B\mu$  is much smaller than  $\mu^2$  for  $M_* \ll M_{\text{Pl}}$ , which is the region we are interested in; see below.

respectively, where  $\mathcal{W}_\alpha$  and  $\mathcal{W}_\alpha^a$  ( $a = 1, 2, 3$ ) are the field-strength superfields for  $G_{\text{hid}}$  and  $G_{\text{SM}}$ ,<sup>3</sup> and  $A_{\text{hid}}$  and  $A_{\text{SM}}$  the  $U(1)_H$ - $G_{\text{hid}}^2$  and  $U(1)_H$ - $G_{\text{SM}}^2$  anomalies. Here, we have normalized the  $U(1)_H$  charge of  $X$  to be 2, and assumed a nonvanishing vacuum expectation value (VEV) of  $X$ , as well as the universality of  $A_{\text{SM}}$  with respect to the three gauge group factors of  $G_{\text{SM}}$ . Note that the form of Eqs. (3, 4) is completely dictated by the anomalous  $U(1)_H$  symmetry: under  $X \rightarrow e^{2i\alpha}X$ , the Lagrangian must transform as  $\mathcal{L} \rightarrow \mathcal{L} - (\alpha/32\pi^2)(A_{\text{hid}}F_{\mu\nu}\tilde{F}^{\mu\nu} + A_{\text{SM}}F_{\mu\nu}^a\tilde{F}^{a\mu\nu})$ , where  $F_{\mu\nu}$  and  $F_{\mu\nu}^a$  are the field strengths for  $G_{\text{hid}}$  and  $G_{\text{SM}}$ . We find that the operator of Eq. (3) is the one needed to generate a superpotential for  $X$  through the hidden sector dynamics, and the operators of Eq. (4) are responsible for the gaugino masses.

The  $U(1)_H$  anomalies with  $G_{\text{hid}}$  and  $G_{\text{SM}}$  can be enforced by coupling  $X$  to a vector-like field(s)  $F, \bar{F}$ :  $W = \lambda X F \bar{F}$  in Eq. (1). If  $F$  and  $\bar{F}$  are charged under  $G_{\text{hid}}$  ( $G_{\text{SM}}$ ), a nonvanishing  $U(1)_H$ - $G_{\text{hid}}^2$  ( $U(1)_H$ - $G_{\text{SM}}^2$ ) anomaly results:  $A_{\text{hid}} = -2T_F^{\text{hid}}$  ( $A_{\text{SM}} = -2T_F^{\text{SM}}$ ), where  $T_F^{\text{hid}}$  ( $T_F^{\text{SM}}$ ) is the Dynkin index of  $F$  under  $G_{\text{hid}}$  ( $G_{\text{SM}}$ ), normalized to be 1/2 for the fundamental representation of  $SU(N)$ . The operators of Eqs. (3, 4) then appear after integrating out the  $F, \bar{F}$  fields. With the interaction Eq. (3), the dynamics of  $G_{\text{hid}}$  generates the effective superpotential

$$W_{\text{eff}} = \Lambda_{\text{eff}}^3 = (\lambda X)^{T_F^{\text{hid}}/T_G^{\text{hid}}} \Lambda^{(3T_G^{\text{hid}} - T_F^{\text{hid}})/T_G^{\text{hid}}}, \quad (5)$$

through gaugino condensation [11]. Here,  $\Lambda_{\text{eff}}$  and  $\Lambda$  are the dynamical scales of the low-energy effective pure  $G_{\text{hid}}$  theory and the original  $G_{\text{hid}}$  theory, respectively, and  $T_G^{\text{hid}}$  is the Dynkin index for the adjoint representation of  $G_{\text{hid}}$  ( $N$  for  $G_{\text{hid}} = SU(N)$ ). We find that for  $T_F^{\text{hid}} = T_G^{\text{hid}}$  the generated superpotential is linear in  $X$ , so that supersymmetry is broken by  $\partial W_{\text{eff}}/\partial X \neq 0$ .

Supersymmetry breaking by the operator of Eq. (5) was considered in Refs. [3, 4] to build models of “direct” gauge mediation. With  $T_F^{\text{hid}} = T_G^{\text{hid}}$ ,  $X$  is a pseudo-flat direction and has a supersymmetry breaking plateau at  $X \gtrsim \Lambda/\lambda$ . (For  $X \lesssim \Lambda/\lambda$  the mass of  $F, \bar{F}$  becomes smaller than the dynamical scale  $\Lambda$ , and our present analysis breaks down.) Now, suppose that  $X$  is stabilized at some value  $\langle X \rangle$ . Integrating out the  $F, \bar{F}$  fields charged under  $G_{\text{SM}}$ , then, generates gauge mediated contributions to the MSSM gaugino and scalar masses of order

$$m_{\text{GMSB}} \approx \frac{g^2}{16\pi^2} \frac{\lambda \Lambda^2}{\langle X \rangle}, \quad (6)$$

where  $g$  represents the standard model gauge couplings [8, 9]. On the other hand, the  $\mu$  parameter generated by Eq. (2) is of order

$$\mu \approx \frac{\lambda \Lambda^2}{M_*}. \quad (7)$$

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<sup>3</sup>The field-strength superfields are normalized such that the gauge kinetic terms are given by  $\mathcal{L}_{\text{kin}} = \int d^2\theta \{ (1/4g^2) \mathcal{W}^\alpha \mathcal{W}_\alpha + \sum_a (1/4g_a^2) \mathcal{W}^{a\alpha} \mathcal{W}_\alpha^a \} + \text{h.c.}$ , where  $g$  and  $g_a$  are the gauge couplings for  $G_{\text{hid}}$  and  $G_{\text{SM}}$ .

How can these two contributions be the same order? In the absence of other sources for the  $\mu$  or supersymmetry breaking parameters, these two contributions must be comparable, which requires  $\langle X \rangle \approx (g^2/16\pi^2)M_* \approx 10^{-2}M_*$ . Is there any reason to expect  $\langle X \rangle$  to be in this particular range?

In fact, one of the simplest ways to stabilize  $X$  gives us such a reason. The existence of the operator Eq. (2) suggests that the  $X$  field also has higher dimension operators in the Kähler potential suppressed by powers of  $M_*$ . Now, suppose that the coefficient of the lowest such operator,  $|X|^4$ , is negative:

$$K = -\frac{1}{4M_*^2}|X|^4, \quad (8)$$

where we have put the factor  $1/4$  to take into account the symmetry factor. This gives a positive mass squared to the pseudo-flat direction  $X$ , since the potential has the contribution  $V \sim |\partial W/\partial X|^2(\partial^2 K/\partial X^\dagger \partial X)^{-1} \supset (|\lambda\Lambda^2|^2/M_*^2)|X|^2$ . The resulting minimum, however, is not at the origin (which would push  $\langle X \rangle$  away from the plateau). This is because in supergravity the potential receives the contribution  $-3|W|^2/M_{\text{Pl}}^2 - \{F_X(\partial K/\partial X)W + \text{h.c.}\}/M_{\text{Pl}}^2$ , leading to a linear term in  $X$  [6]:  $V \sim \lambda\Lambda^2 c(X + X^\dagger)/M_{\text{Pl}}^2$ , where  $c$  is the constant term in the superpotential needed to cancel the cosmological constant. Balancing these two effects and setting  $c \sim \lambda\Lambda^2 M_{\text{Pl}}$  to cancel the positive vacuum energy of the plateau leads to

$$\langle X \rangle \approx \frac{M_*^2}{M_{\text{Pl}}}. \quad (9)$$

This gives the desired relation  $\langle X \rangle \approx 10^{-2}M_*$  if we choose  $M_*$  to be the unification scale,  $M_* \simeq 10^{16}$  GeV, one of the natural choices for the cutoff of the MSSM. This coincidence of scales was noticed in Ref. [10], where the effective field theory description/parameterization of the class of dynamics under consideration was also discussed in detail.

To stabilize  $X$  at the value of Eq. (9) and make the supersymmetry breaking masses and  $\mu$  comparable, the dominant deformation of the plateau must come from the two effects described above. This gives a restriction on the range of the parameter  $\lambda$ , appearing in Eq. (1). First, the effect from  $F, \bar{F}$  loops on the  $X$  potential must be subdominant. This gives an upper bound on  $\lambda$ . The stabilized value of  $\langle X \rangle$  must also satisfy  $M_F \equiv \lambda\langle X \rangle > \Lambda$ , giving a lower bound on  $\lambda$ . We study these and other bounds on  $\lambda$ , and find that there is a region where all the requirements are satisfied. This, together with Eq. (9), is then translated into the allowed range for the mass of the  $F, \bar{F}$  fields,  $M_F$ .

The arguments described above lead to the following picture for supersymmetric theories in which the questions listed in section 1 are addressed:

- The effective cutoff scale of the MSSM is the unification scale,  $M_* \approx 10^{16}$  GeV. The Higgs and supersymmetry breaking fields have higher dimension operators suppressed by powers

of  $M_*$ . This is consistent with successful gauge coupling unification in supersymmetric theories.

- There is no tree-level operator connecting matter and supersymmetry breaking fields in the Kähler potential  $K \approx M^\dagger M X^\dagger X / M_*^2$ , where  $M$  represents the MSSM matter fields. Such operators would generically lead to large flavor changing neutral currents at low energies, and so should be suppressed. This property must arise from the theory at or above the effective cutoff scale  $M_*$ .
- The supersymmetry breaking field  $X$  is charged under  $U(1)_H$  which has anomalies with respect to both  $G_{\text{hid}}$  and  $G_{\text{SM}}$ . The anomalies are enforced by coupling  $X$  to a vector-like pair(s) of fields  $F$  and  $\bar{F}$  in the superpotential,  $W = \lambda X F \bar{F}$ . The simplest possibility to make both the  $U(1)_H$ - $G_{\text{hid}}^2$  and  $U(1)_H$ - $G_{\text{SM}}^2$  anomalies nonvanishing is to consider  $F, \bar{F}$  to be charged under both  $G_{\text{hid}}$  and  $G_{\text{SM}}$ .
- To generate the effective superpotential linear in  $X$  by the hidden sector dynamics, the Dynkin index of  $F, \bar{F}$  under  $G_{\text{hid}}$  must be the same as that of the adjoint representation of  $G_{\text{hid}}$ :  $T_F^{\text{hid}} = T_{\bar{F}}^{\text{hid}}$ . The simplest possibility to realize this is to consider that  $G_{\text{hid}} = SU(5)_{\text{hid}}$ , and that  $F, \bar{F}$  are “bi-fundamental” under  $SU(5)_{\text{hid}} \times SU(5)_{\text{SM}}$ :  $F(\mathbf{5}, \mathbf{5}^*)$  and  $\bar{F}(\mathbf{5}^*, \mathbf{5})$ , where  $SU(5)_{\text{SM}} \supset G_{\text{SM}}$ .

In the next two sections, we present an explicit model based on these observations. The analysis of the dynamics of the model using effective field theory will be given in section 5.

### 3 Model

The gauge group of the model is  $SU(5)_{\text{hid}} \times G_{\text{SM}}$ . In addition to the MSSM fields,  $Q, U, D, L, E, H_u$  and  $H_d$ , which are all singlet under  $SU(5)_{\text{hid}}$ , we introduce a singlet field and a pair of “bi-fundamental” fields:

$$X(\mathbf{1}, \mathbf{1}), \quad F(\mathbf{5}, \mathbf{5}^*), \quad \bar{F}(\mathbf{5}^*, \mathbf{5}), \quad (10)$$

where the numbers in parentheses represent quantum numbers under  $SU(5)_{\text{hid}} \times SU(5)_{\text{SM}}$ . Here, we have used the language of  $SU(5)_{\text{SM}} \supset G_{\text{SM}}$  for simplicity of notation, but  $G_{\text{SM}}$  does not have to be unified into a single gauge group.

We now introduce a (anomalous)  $U(1)$  global symmetry,  $U(1)_H$ , under which the fields transform as

$$X(2), \quad F(-1), \quad \bar{F}(-1), \quad (11)$$

$$Q(x), \quad U(-1-x), \quad D(-1-x), \quad L(y), \quad E(-1-y), \quad (12)$$

$$H_u(1), \quad H_d(1), \quad (13)$$

where  $x$  and  $y$  are some numbers. The charges of  $H_u$  and  $H_d$  are determined so that the term  $X^\dagger H_u H_d$  is allowed in the Kähler potential (we can use a hypercharge rotation to make the  $H_u$  and  $H_d$  charges equal without loss of generality), and those of matter are determined so that the Yukawa couplings are invariant under  $U(1)_H$ .

We take the cutoff scale of our theory to be around the unification scale,  $M_* \simeq 10^{16}$  GeV. This preserves successful gauge coupling unification. The most general Kähler potential and superpotential among the  $X$ ,  $F$  and  $\bar{F}$  fields consistent with  $SU(5)_{\text{hid}} \times G_{\text{SM}} \times U(1)_H$  are then

$$K = K_{\text{kin}} - \frac{1}{4M_*^2}(X^\dagger X)^2 + \dots, \quad (14)$$

$$W = c + \lambda X F \bar{F} + \dots, \quad (15)$$

where  $K_{\text{kin}}$  represents the canonically normalized kinetic terms,  $c$  is a constant term in the superpotential, and  $\lambda$  is a coupling constant. Here, we have assumed that the coefficient of the second term in Eq. (14) is negative, and absorbed its magnitude into the definition of  $M_*$ . The parameters  $c$  and  $\lambda$  are taken to be real and positive without loss of generality by using  $U(1)_R$  and  $F\bar{F}$  rotations. The most general interactions between  $X$ ,  $F$ ,  $\bar{F}$  and the Higgs fields are

$$K \approx \left( \frac{1}{M_*} X^\dagger H_u H_d + \text{h.c.} \right) + \frac{1}{M_*^2} X^\dagger X H_u^\dagger H_u + \frac{1}{M_*^2} X^\dagger X H_d^\dagger H_d + \dots, \quad (16)$$

$$W = \frac{\eta}{M_*} F \bar{F} H_u H_d + \dots, \quad (17)$$

where  $\eta$  is a dimensionless coupling. Note that we have taken the theory to be weakly coupled at  $M_*$  (or strongly coupled at  $\approx 4\pi M_*$ ), so that the dimensionless coefficients in the Kähler potential, omitted in Eq. (16), are naturally of order unity. On the other hand, the superpotential couplings can be naturally smaller because they are radiatively stable. We assume the absence of interactions between the  $X$  and matter fields suppressed by powers of  $M_*$ , as stated in the previous section. This can be achieved, for example, if the  $X$  and matter fields are localized to distant points in (small) extra dimensions, with the Higgs fields propagating in the bulk.

We now demonstrate that the simple model described above gives successful supersymmetry breaking and its mediation. We first consider the minimization of the potential for  $X$ . At low energies, the fields  $F$  and  $\bar{F}$  decouple at the mass  $M_F = \lambda X$ , and the gaugino condensation of  $SU(5)_{\text{hid}}$  generates the superpotential  $W = \Lambda_{\text{eff}}^3$ . Here,  $\Lambda_{\text{eff}}$  is the effective dynamical scale for the low-energy pure  $SU(5)$  gauge theory, which is related to the dynamical scale  $\Lambda$  of the original  $SU(5)_{\text{hid}}$  by the matching condition  $\Lambda_{\text{eff}}^3 = M_F \Lambda^2 = \lambda X \Lambda^2$ .<sup>4</sup> This implies that the dynamics of  $SU(5)_{\text{hid}}$  generates the superpotential

$$W_{\text{eff}} = \lambda \Lambda^2 X, \quad (18)$$

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<sup>4</sup>The precise definition of  $\Lambda$  here is given such that the generated superpotential is  $W = \Lambda_{\text{eff}}^3$ .



which leads to a supersymmetry breaking plateau for the  $X$  potential, taking the form  $V \sim |\lambda\Lambda^2|^2$  in the limit that we neglect supergravity and higher order corrections. Note that this analysis is valid only for  $|M_F| = |\lambda X| \gtrsim \Lambda$ . For  $|\lambda X| \lesssim \Lambda$ ,  $SU(5)_{\text{hid}}$  has 5 flavors of light “quarks,” and there are supersymmetric minima at  $X = 0$  even at the quantum level.<sup>5</sup>

The supersymmetry breaking plateau at  $|X| \gtrsim \Lambda/\lambda$  is distorted by a number of corrections, including effects from higher dimension operators in Eq. (14), supergravity terms, and loops of the  $F$  and  $\bar{F}$  fields. With the negative sign for its coefficient, the second term in Eq. (14) induces a positive mass-squared term for  $X$  in the potential,  $\delta V = (\lambda^2\Lambda^4/M_*^2)|X|^2$ . On the other hand, supergravity corrections lead to a linear term in  $X$ ,  $\delta V = -2\lambda\Lambda^2 c(X + X^\dagger)/M_{\text{Pl}}^2$ .<sup>6</sup> These two effects lead to a minimum at  $\langle X \rangle = 2cM_*^2/\lambda\Lambda^2 M_{\text{Pl}}^2$ . The value of  $c$  is determined by the condition of a vanishing cosmological constant at this minimum in the supergravity potential,  $V \simeq \lambda^2\Lambda^4 - 3c^2/M_{\text{Pl}}^2 = 0$ . This leads to  $c \simeq \lambda\Lambda^2 M_{\text{Pl}}/\sqrt{3}$ , and hence

$$\langle X \rangle \simeq \frac{2M_*^2}{\sqrt{3}M_{\text{Pl}}} \approx 10^{14} \text{ GeV}, \quad F_X \simeq -\lambda\Lambda^2, \quad (19)$$

where  $F_X \equiv \langle -\partial W^\dagger/\partial X^\dagger - (\partial K/\partial X^\dagger)W^\dagger/M_{\text{Pl}}^2 \rangle$  is the supersymmetry breaking VEV for the  $X$  superfield. Note that  $\langle X \rangle \approx M_*(M_*/M_{\text{Pl}}) < M_*$ , so that it stays within the regime where the effective field theory below  $M_*$  is applicable.

The loops of the  $F, \bar{F}$  fields also affect the  $X$  potential. One such effect arises from the 1-loop Coleman-Weinberg correction to the Kähler potential  $\delta K_{\text{CW}} \simeq -(\lambda^2 n_F/16\pi^2)|X|^2 \ln(|\lambda X|^2/\mu_R^2)$ , where  $n_F \equiv n_G^2 = 25$  is the number of degrees of freedom for  $F$ , and  $\mu_R$  the renormalization scale. This effect is small enough not to destabilize the minimum of Eq. (19), as long as

$$\frac{\lambda^2 n_G^2}{16\pi^2} \lesssim \left(\frac{M_*}{M_{\text{Pl}}}\right)^2 \approx 10^{-4}, \quad (20)$$

where  $\lambda$  is evaluated at the scale  $\approx \lambda\langle X \rangle$ . Another effect arises from the generation of the Kähler potential operator  $\delta K_{\mathcal{W}^4} \approx (1/64)(N_{\text{SM}}/16\pi^2)|\mathcal{W}^\alpha \mathcal{W}_\alpha|^2/|\lambda X|^4$  with  $\mathcal{W}^\alpha \mathcal{W}_\alpha \rightarrow 32\pi^2 \Lambda_{\text{eff}}^3/N_{\text{hid}} = 32\pi^2 \lambda X \Lambda^2/N_{\text{hid}}$ , where  $\mathcal{W}_\alpha$  is the field-strength superfield for  $SU(5)_{\text{hid}}$ , and  $N_{\text{hid}} = N_{\text{SM}} = 5$  are the number of “colors” for  $SU(5)_{\text{hid}}$  and  $SU(5)_{\text{SM}}$  (1/64 is the symmetry factor). This effect is unimportant as long as

$$\lambda^2 \gtrsim \frac{\pi M_{\text{Pl}}^3 |F_X|}{\sqrt{n_G} M_*^5}, \quad (21)$$

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<sup>5</sup>The minima are at  $X = 0$ ,  $\text{tr}(M_j^i) = 0$  and  $\det(M_j^i) - B\bar{B} = (\Lambda^2/5)^5$ , where  $M_j^i \equiv F_\alpha^i \bar{F}_j^\alpha$ ,  $B \equiv \epsilon^{\alpha\beta\gamma\delta\eta} F_\alpha^i F_\beta^j F_\gamma^k F_\delta^l F_\eta^m \epsilon_{ijklm}$  and  $\bar{B} \equiv \epsilon_{\alpha\beta\gamma\delta\eta} \bar{F}_i^\alpha \bar{F}_j^\beta \bar{F}_k^\gamma \bar{F}_l^\delta \bar{F}_m^\eta \epsilon^{ijklm}$  are the “meson,” “baryon” and “antibaryon” superfields of  $SU(5)_{\text{hid}}$ , with  $\alpha, \beta, \dots$  and  $i, j, \dots$  representing the indices of  $SU(5)_{\text{hid}}$  and  $SU(5)_{\text{SM}}$ , respectively.

<sup>6</sup>In the superconformal calculus formulation of supergravity [12], the  $X$  linear term in the scalar potential arises from the  $F$ -term VEV,  $F_\varphi = c/M_{\text{Pl}}^2$ , of the chiral compensator field  $\varphi$  through the superpotential term  $W = \varphi^2 \lambda X \Lambda^2$ . From the viewpoint of the original theory Eqs. (14 – 17), this effect is understood to arise from the anomaly-mediated contribution [13] to the  $SU(5)_{\text{hid}}$  gaugino mass.

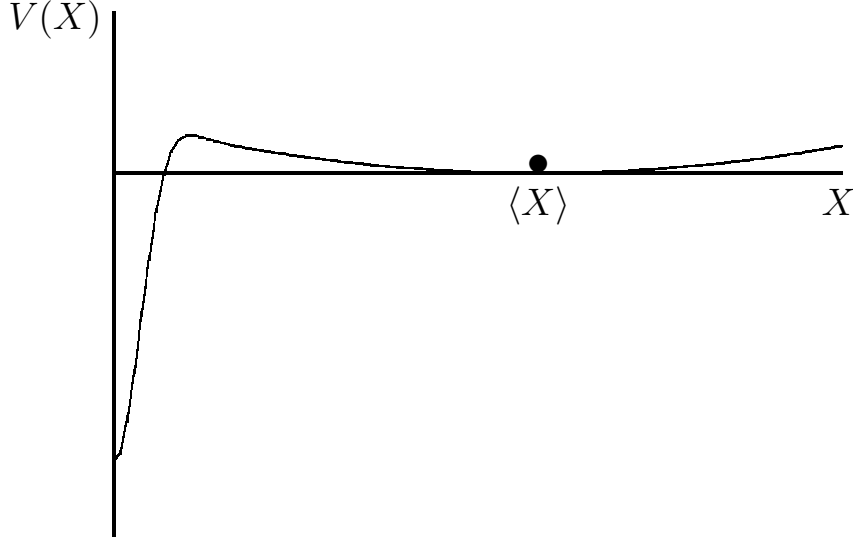


Figure 1: The schematic depiction of the  $X$  potential.

where  $n_G = 5$ . There is also a correction arising from nonperturbative effects of  $SU(5)_{\text{hid}}$ , given by  $\delta K_{\text{np}} \approx k|\Lambda_{\text{eff}}|^2 \approx k|\lambda\Lambda^2 X|^{2/3}$ , where  $k \approx (N_{\text{hid}}^2/16\pi^2)^{1/3} \simeq O(1)$  [14]. This correction is irrelevant if

$$\frac{|F_X|M_{\text{Pl}}^5}{M_*^7} \lesssim 10^2, \quad (22)$$

where we have used Eq. (19). We assume that these conditions are satisfied, so that the minimum of  $X$  is given by Eq. (19). There is one remaining condition on  $\lambda$  which comes from the requirement that the minimum lies on the plateau,  $|\langle X \rangle| \gtrsim (4\pi/\sqrt{N_{\text{SM}}N_{\text{hid}}})\Lambda/\lambda$ . Here, we have included the factor of  $4\pi/\sqrt{N_{\text{SM}}}$  suggested by naive dimensional analysis [15], and the factor  $1/\sqrt{N_{\text{hid}}}$  arises from our definition of  $\Lambda$  (see footnote 4). This gives

$$\lambda^3 \gtrsim \frac{16\pi^2 M_{\text{Pl}}^2 |F_X|}{n_G^2 M_*^4}. \quad (23)$$

One implication of these conditions, Eqs. (20 – 23), will be discussed later.

The potential for  $X$  in our model is depicted schematically in Fig. 1. We are living in a very flat supersymmetry breaking plateau with  $V \sim 0$ . The tunneling rate from our local minimum to the true (supersymmetric) minimum can be estimated using the technique of Ref. [16]. The decay rate per unit volume is given by  $\Gamma/V \sim \langle X \rangle^4 e^{-B}$ , where  $B \sim 2\pi^2 \langle X \rangle^4 / F_X^2 \sim 10^{20}$ . Here, we have used Eq. (31) below to obtain the numerical estimate for  $B$ . We find that the lifetime of the local minimum is much larger than the age of the universe:  $\Gamma/V \ll H_0^4$ , where  $H_0 \simeq 10^{-33}$  eV

is the present Hubble constant. The mass squared for  $X$  around the minimum is given by

$$m_X^2 \simeq \left( \frac{F_X}{M_*} \right)^2. \quad (24)$$

As we will see, this is of the order of the weak scale squared.

## 4 Superparticle Masses

With the  $X$  VEV and  $F_X$  in Eq. (19), the supersymmetry breaking and  $\mu$  parameters in the MSSM receive several contributions. First, the interactions in Eq. (16) give

$$\mu \approx \frac{F_X}{M_*}, \quad m_{H_u}^2 \approx m_{H_d}^2 \approx \left( \frac{F_X}{M_*} \right)^2, \quad (25)$$

at the scale  $M_*$ . Here,  $m_{H_u}^2$  and  $m_{H_d}^2$  are the non-holomorphic supersymmetry breaking squared masses for  $H_u$  and  $H_d$ . On the other hand, the interaction of Eq. (17) gives  $\mu \approx \eta \Lambda^2 / M_* \approx (\eta/\lambda) F_X / M_*$ . (This is obtained by making the replacement  $\lambda X \rightarrow \lambda X + \eta H_u H_d / M_*$  in Eq. (18).) We assume  $\eta \lesssim \lambda$  so that this contribution is, at most, comparable to that in Eq. (25).<sup>7</sup> The masses generated at the scale  $M_*$  then take the form of Eq. (25), with the other supersymmetry breaking parameters essentially vanishing. Note that here we have omitted  $O(1)$  coefficients, so that the ratio of  $m_{H_u}^2$  to  $m_{H_d}^2$ , for example, can be an arbitrary  $O(1)$  number.

The supersymmetry breaking masses also receive contributions from loops of the  $F$ ,  $\bar{F}$  fields through gauge mediation, generated at the mass scale of these fields

$$M_{\text{mess}} \approx \lambda \langle X \rangle \approx \frac{\lambda M_*^2}{M_{\text{Pl}}}. \quad (26)$$

The gaugino masses  $M_a$  ( $a = 1, 2, 3$ ) and the scalar squared masses  $m_{\tilde{f}}^2$  ( $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}, \tilde{e}$ ) receive contributions

$$M_a = N_{\text{mess}} \frac{g_a^2}{16\pi^2} \frac{F_X}{\langle X \rangle}, \quad (27)$$

$$m_{\tilde{f}}^2 = 2N_{\text{mess}} \sum_a C_a^{\tilde{f}} \left( \frac{g_a^2}{16\pi^2} \right)^2 \left| \frac{F_X}{\langle X \rangle} \right|^2, \quad (28)$$

where  $a = 1, 2, 3$  represents the standard model gauge group factors,  $g_a$  the standard model gauge couplings at  $M_{\text{mess}}$ , and  $C_a^{\tilde{f}}$  the group theory factors given by  $(C_1^{\tilde{f}}, C_2^{\tilde{f}}, C_3^{\tilde{f}}) = (1/60, 3/4, 4/3)$ ,  $(4/15, 0, 4/3)$ ,  $(1/15, 0, 4/3)$ ,  $(3/20, 3/4, 0)$  and  $(3/5, 0, 0)$  for  $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}$  and  $\tilde{e}$ , respectively. The

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<sup>7</sup>This is easily achieved, for example, if the suppressions of  $\lambda$  and  $\eta$  have a common origin, such as the suppression of the couplings between the  $F$ ,  $\bar{F}$  fields and the rest of the fields (in which case we expect  $\lambda \approx \eta$ ).

contributions to  $m_{H_u}^2$  and  $m_{H_d}^2$  are the same as that to  $m_t^2$ . The quantity  $N_{\text{mess}}$  is the number of messenger pairs, which is predicted as

$$N_{\text{mess}} = 5, \quad (29)$$

in the present model.

The low-energy superparticle masses are obtained by evolving the parameters of Eq. (25) from  $M_*$  to  $M_{\text{mess}}$ , adding the contributions of Eqs. (27, 28) at  $M_{\text{mess}}$ , and then evolving the resulting parameters from  $M_{\text{mess}}$  down to the weak scale. Note that since the gauge-mediated contributions of Eqs. (27, 28) have the size

$$M_a \approx (m_{\tilde{f}}^2)^{1/2} \approx \frac{F_X}{M_*} \left( \frac{g^2}{16\pi^2} \frac{M_{\text{Pl}}}{M_*} \right) \approx \frac{F_X}{M_*}, \quad (30)$$

where  $g$  represents the standard model gauge couplings, they are comparable to the tree-level contributions to the Higgs-sector parameters of Eq. (25).<sup>8</sup> Setting the size of these contributions to be the weak scale

$$\left| \frac{F_X}{M_*} \right| \approx (100 \text{ GeV} - 1 \text{ TeV}), \quad (31)$$

the value of  $F_X$  is determined as  $\sqrt{F_X} \approx (10^9 - 10^{9.5}) \text{ GeV}$ . With these values of  $F_X$ , the condition of Eq. (22) is satisfied. The messenger scale of gauge mediation,  $M_{\text{mess}}$ , is given by Eq. (26), and is subject to the bounds on  $\lambda$  in Eqs. (20), (21) and (23), which can be written as  $\lambda^2 \lesssim 10^{-3}$ ,  $\lambda^2 \gtrsim 10^{-8}$  and  $\lambda^3 \gtrsim 10^{-9}$  using Eq. (31). These lead to the following range on the messenger scale of gauge mediation:

$$10^{11} \text{ GeV} \lesssim M_{\text{mess}} \lesssim 10^{13} \text{ GeV}. \quad (32)$$

Here, we have taken into account the existence of possible  $O(1)$  factors to derive these numbers.

There are only two nontrivial phases appearing in the superparticle masses: the phase of  $\mu$  in Eq. (25) and that of the gaugino masses in Eq. (27). These can be absorbed into the phases of the fields using  $R$  and Peccei-Quinn rotations, so that the supersymmetric  $CP$  problem is absent. The squark and slepton masses receive dominant contributions from gauge mediation, which are flavor universal and thus do not lead to the supersymmetric flavor problem. On the other hand, they also receive renormalization group contributions from  $m_{H_u}^2$  and  $m_{H_d}^2$  through the Yukawa couplings in the energy interval between  $M_*$  and the weak scale. This may lead to nontrivial flavor violating processes at a level consistent with but close to the current experimental bounds. The spectrum of the superparticles is essentially that of gauge mediation, although the third

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<sup>8</sup>In contrast with the situation discussed in Ref. [10], there is no reason in our theory why the  $\mu$  term must be suppressed compared with the gauge-mediated contributions. In fact, they are naturally expected to be comparable.

generation superparticle masses, as well as the Higgs soft masses, can have significant deviations from it due to renormalization group contributions from  $m_{H_u}^2$  and  $m_{H_d}^2$ . These deviations are parameterized by two free parameters of the model:  $m_{H_u}^2$  and  $m_{H_d}^2$  at  $M_*$ . The predictions of Eqs. (29, 32), however, can be tested without taking into account these effects by using the first two generation superparticle masses. Note that there is no correction from the hidden sector dynamics [17], since the  $X$  field is extremely weakly coupled below the messenger scale  $M_{\text{mess}}$ .

Independently from the parameters of the model, the gravitino mass is given by

$$m_{3/2} \simeq \frac{F_X}{\sqrt{3}M_{\text{Pl}}} \approx (1 - 10) \text{ GeV}, \quad (33)$$

implying that the gravitino is the lightest supersymmetric particle. The next-to-lightest supersymmetric particle, which is mostly the right-handed stau, then decays into the gravitino with a lifetime of order  $\tau_{\tilde{\tau}} \simeq 48\pi m_{3/2}^2 M_{\text{Pl}}^2 / m_{\tilde{\tau}}^5 \approx (10^2 - 10^6) \text{ sec}$ . This leads to interesting phenomenology at future collider experiments.

## 5 Effective Field Theory Analysis

The analysis performed in the previous sections does not strictly follow the method of effective field theory, in which heavy degrees of freedom are integrated out in discussing the dynamics of low energy excitations. Here we discuss the dynamics of the model using effective field theories.

For large values of  $X$ ,  $|X| \gtrsim \Lambda/\lambda$ , the largest physical scale below the cutoff  $M_*$  is the mass of the  $F$ ,  $\bar{F}$  fields,  $|M_F| = |\lambda X|$ . Below this scale, the  $F$ ,  $\bar{F}$  fields are integrated out, leaving an effective field theory that contains only the  $SU(5)_{\text{hid}}$  gauge supermultiplet, MSSM fields, and  $X$ . The integration generates several important operators. First, it generates

$$\mathcal{L} = \left\{ - \int d^2\theta \sum_a \frac{n_G}{32\pi^2} \left( \ln \frac{X}{|\langle X \rangle|} \right) \mathcal{W}_a^\alpha \mathcal{W}_{a\alpha} + \text{h.c.} \right\} - \int d^4\theta \sum_\Phi \sum_a \frac{g_a^4 n_G}{(16\pi^2)^2} C_a^\Phi \left( \ln \frac{X^\dagger X}{|\langle X \rangle|^2} \right)^2 \Phi^\dagger \Phi, \quad (34)$$

where  $\mathcal{W}_a^\alpha$  ( $a = 1, 2, 3$ ) are the field-strength superfields for  $G_{\text{SM}}$ , and  $\Phi = Q, U, D, L, E, H_u, H_d$  are the MSSM matter and Higgs superfields. The couplings  $g_a$  are evaluated at  $|\lambda \langle X \rangle|$ , and  $\langle X \rangle$  will be determined by minimizing the potential in the low energy theory. These operators become gauge-mediated gaugino and scalar masses when  $\langle X \rangle \neq 0$  and  $F_X \neq 0$  [18]. The operator

$$\mathcal{L} = - \int d^2\theta \frac{n_G}{32\pi^2} \left( \ln \frac{X}{|\langle X \rangle|} \right) \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.}, \quad (35)$$

is also generated, where  $\mathcal{W}_\alpha$  is the field-strength superfield for  $SU(5)_{\text{hid}}$ . Here, we have neglected the contribution from Eq. (17), assuming that  $\eta$  is sufficiently small ( $\eta \lesssim \lambda$ ) so that it gives only

a phenomenologically irrelevant effect. Note that the first terms of Eq. (34) and Eq. (35) are the manifestations of  $U(1)_H$  mixed anomalies with  $G_{\text{SM}}$  and  $SU(5)_{\text{hid}}$ , respectively.

Integrating out  $F, \bar{F}$  also generates corrections to the Kähler potential containing  $X$  and  $\mathcal{W}_\alpha$ . Together with the tree-level terms in Eq. (14), the Kähler potential for  $X$  and  $\mathcal{W}_\alpha$  is given by

$$K = X^\dagger X - \frac{1}{4M_*^2}(X^\dagger X)^2 + \delta K_{\text{CW}} + \delta K_{\mathcal{W}^4} + \dots, \quad (36)$$

where  $\delta K_{\text{CW}}$  and  $\delta K_{\mathcal{W}^4}$  are given above Eqs. (20) and (21), respectively. (These terms are not important for the dynamics of the model in the parameter region we are interested.) The effective theory below  $|\lambda\langle X \rangle|$  is then given by Eqs. (34, 35, 36), together with Eq. (16), the MSSM kinetic and Yukawa terms, the gauge kinetic term for  $SU(5)_{\text{hid}}$ , and the constant term in the superpotential. Since  $X$  has only irrelevant interactions below  $|\lambda\langle X \rangle|$ , the operators of Eqs. (16, 34) run only by loops of the MSSM states. As a result, renormalization group evolutions for  $\mu$  and the supersymmetry breaking masses are exactly those of the MSSM below the messenger scale  $M_{\text{mess}} = |\lambda\langle X \rangle|$ .

At the scale  $|\Lambda_{\text{eff}}| = |\lambda X \Lambda^2|^{1/3}$ ,  $SU(5)_{\text{hid}}$  gauge interactions become strong, giving nonperturbative effects. The  $SU(5)_{\text{hid}}$  gauge multiplet should be integrated out. In particular, the  $SU(5)_{\text{hid}}$  gauge kinetic term and Eq. (35) are replaced by the superpotential term of  $\Lambda_{\text{eff}}^3$ , leading to the superpotential

$$W = \lambda X \Lambda^2 + c, \quad (37)$$

in the effective theory below  $\Lambda_{\text{eff}}$ . (The MSSM Yukawa terms should also exist.) The combination  $\mathcal{W}^\alpha \mathcal{W}_\alpha$  in Eq. (36) is also replaced by the condensation  $\langle \mathcal{W}^\alpha \mathcal{W}_\alpha \rangle = 32\pi^2 \lambda X \Lambda^2 / n_G$ , and the Kähler potential term

$$\delta K_{\text{np}} \approx |\Lambda_{\text{eff}}|^2 \approx |\lambda \Lambda^2 X|^{2/3}, \quad (38)$$

is generated. The theory now contains only the MSSM states and  $X$ , whose interactions are given by Eqs. (16, 34, 36, 37, 38) and the MSSM Yukawa couplings.

Below  $|\Lambda_{\text{eff}}| = |\lambda X \Lambda^2|^{1/3}$ , the dynamics of  $X$  decouple from the rest, so that the minimum of the potential,  $\langle X \rangle$ , is determined by Eqs. (36, 37, 38). In order for the analysis to be consistent, the resulting  $\langle X \rangle$  should satisfy  $\Lambda/\lambda \lesssim |\langle X \rangle| \lesssim M_*$ . In fact, for  $10^{-6} \lesssim \lambda^2 \lesssim 10^{-3}$ , the minimum is given by Eq. (19) and is within this range. It is instructive to write  $\langle X \rangle$  in the form

$$\langle X \rangle = \frac{2c(\lambda \Lambda^2)^\dagger}{|\lambda \Lambda^2|^2} \frac{M_*^2}{M_{\text{Pl}}^2}, \quad (39)$$

although  $\lambda \Lambda^2$  can be chosen real, and  $c$  is set to  $c = \lambda \Lambda^2 M_{\text{Pl}} / \sqrt{3}$  by the condition of vanishing cosmological constant. This shows why  $X$  can obtain a nonzero VEV despite the fact that it is charged under both  $U(1)_H$  and an accidental  $U(1)_R$  symmetry possessed by the  $\lambda$  coupling:

$R(X) = 2$ ,  $R(F) = R(\bar{F}) = 0$ . The  $U(1)_H$  symmetry is broken by the anomaly, i.e.,  $\Lambda$  has a charge of  $-1$ , and  $U(1)_R$  by the constant term in the superpotential, i.e.,  $c$  has a charge of  $+2$ . The expression of Eq. (39) respects both of these spurious symmetries. The mass of  $X$  is given by Eq. (24), which is much smaller than  $\Lambda_{\text{eff}}$ . Minimizing the potential in this low energy effective theory, therefore, is appropriate.

Finally, the expectation values of Eq. (19) give  $\mu$  and the supersymmetry breaking masses through the operators Eqs. (16, 34). The coefficients of the operators in Eq. (16) (Eq. (34)) are subject to renormalization group evolution from  $M_*$  ( $|\lambda\langle X \rangle|$ ) to the scale of the superparticle masses caused by loops of the MSSM states. The loop effects from  $X, F, \bar{F}$  on Eq. (16) between  $M_*$  and  $|\lambda\langle X \rangle|$  are negligible because of the small value of  $\lambda$ .

## 6 More on $U(1)_H$

The  $U(1)_H$  charge assignment of Eqs. (11 – 13) contains two free parameters  $x$  and  $y$ . These parameters can be restricted by imposing various phenomenological requirements. For example, if we require that dangerous dimension-five proton decay operators  $W \sim QQQ L$  and  $UUDE$  are prohibited by  $U(1)_H$ , then we obtain the conditions  $3x + y \neq 0$  and  $3x + y \neq -4$ , respectively. Similarly, if we require that  $U(1)_H$  forbids dimension-four  $R$ -parity violating operators  $W \sim LH_u, QDL, UDD, LLE$  and  $K \sim L^\dagger H_d$ , we obtain  $y \neq -1$ ,  $y \neq 1$ ,  $x \neq -1$ ,  $y \neq 1$  and  $y \neq 1$ . If the values of  $x$  and  $y$  satisfy these conditions, therefore, sufficient proton stability is ensured.<sup>9</sup> The charge assignment of  $x = y = -1/2$  was discussed in Ref. [10].

To generate light neutrino masses, we can introduce three generations of right-handed neutrino superfields  $N$  with the Yukawa couplings  $W \sim LN H_u$ . This determines their  $U(1)_H$  charges to be

$$N(-1 - y). \quad (40)$$

An interesting possibility arises if  $y = 0$ . In this case the superpotential can have interactions of the form  $XN^2$ , so that the right-handed neutrinos can have the superpotential

$$W = \frac{\kappa}{2} X N^2 + y_\nu L N H_u, \quad (41)$$

where  $\kappa$  and  $y_\nu$  are  $3 \times 3$  matrices in generation space. With the  $X$  VEV of Eq. (19), this generates small neutrino masses through the seesaw mechanism. Note that the vacuum of Eq. (19) is not

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<sup>9</sup>Proton decay at a dangerous level may still be caused by operators  $W \sim X^m QQQ L$  and/or  $X^m UUDE$  ( $m \geq 1$ ) through the  $X$  VEV. The absence of these operators, however, is consistent with the absence of the operators  $K \sim X^\dagger X M^\dagger M$  ( $M = Q, U, D, L, E$ ), which we have assumed. Alternatively, the coefficients of these operators, even if they exist, may be suppressed by the corresponding Yukawa coupling factors, in which case the proton is sufficiently stable. Yet another possibility is to impose  $3x + y \neq -8, -6, -4, -2, 0, 2, 4$ , suppressing all the operators  $W \sim X^m QQQ L, X^m UUDE$  with  $m \leq 4$ .

destabilized if  $\kappa \lesssim O(0.1)$ , as can be seen from Eq. (20) with  $\lambda \rightarrow \kappa$  and  $n_G^2 \rightarrow 3$ .

A  $U(1)_H$  charge assignment that satisfies all the requirements above can be obtained with

$$x = \frac{4}{3} + 2n, \quad y = 0, \quad (42)$$

where  $n$  is an integer. The  $U(1)_H$  symmetry is spontaneously broken by the VEV of  $X$ . The charge assignment of Eq. (42), however, leaves a discrete  $Z_6$  symmetry after the breaking. The product of  $Z_6$  and  $U(1)_Y$  contains the (anomalous)  $Z_3$  baryon number and (anomaly-free)  $Z_2$  matter parity ( $R$  parity) as subgroups. This symmetry thus strictly forbids the  $R$ -parity violating operators, and the lightest supersymmetric particle is absolutely stable.

We note that the requirements on  $U(1)_H$  discussed above are not a necessity. For example, the  $R$ -parity violating operators may be forbidden by imposing a matter (or  $R$ ) parity in addition to  $U(1)_H$ , and the dimension-five proton decay operators may be suppressed by some mechanism in the ultraviolet theory. Small neutrino masses may also be obtained by the seesaw mechanism with  $y = -1$  or if there is additional  $U(1)_H$  breaking, or they may simply arise from small Yukawa couplings without the Majorana mass terms for  $N$ . Nevertheless, it is interesting that  $U(1)_H$  can be used to address these issues.<sup>10</sup>

We finally discuss possible origins of  $U(1)_H$ . Since any global symmetry is expected to be broken by quantum gravity effects, we may expect that the ultimate origin of  $U(1)_H$  is a gauge symmetry. One possibility is that  $U(1)_H$  is a remnant of a pseudo-anomalous  $U(1)$  gauge symmetry arising in string theory. Another possibility is that the  $U(1)_H$  symmetry is a gauge symmetry in higher dimensional spacetime broken on a “distant brane.” The superfields of Eqs. (11, 12, 13, 40) are localized on some brane, while the  $G_{\text{SM}}$  and  $G_{\text{hid}}$  gauge fields propagate in the extra dimensional bulk (which we assume to be small). The  $U(1)_H$ - $G_{\text{SM}}$  and  $U(1)_H$ - $G_{\text{hid}}$  anomalies of the original  $U(1)_H$  gauge symmetry are canceled by particles  $\psi$  on the “distant brane,” which become massive through  $U(1)_H$  breaking there. This effectively leaves an anomalous global  $U(1)_H$  symmetry on “our brane.” The scale of  $U(1)_H$  breaking  $v_{\text{dist}}$  on the distant brane should be higher than  $M_F = \lambda \langle X \rangle$  so that the theory is reduced to the model presented here at low energies. If  $v_{\text{dist}} \lesssim M_*$ , we have extra vector-like states  $\psi$  with masses of order  $v_{\text{dist}}$ , which are charged under  $G_{\text{SM}} \times G_{\text{hid}}$  but have no superpotential interactions with the other fields of the model. The existence of such states, however, does not destroy successful gauge coupling unification (as long as they are sufficiently heavy), since they are in complete representations of  $SU(5)_{\text{SM}} \supset G_{\text{SM}}$ .

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<sup>10</sup>An interesting possibility is to assign  $U(1)_H$  charges that depend on the generations, which allows us to use  $U(1)_H$  also as a  $U(1)$  flavor symmetry. For example, we can assign the charges so that the light generation Yukawa couplings arise only through powers of the  $X$  VEV, e.g.  $W \sim \langle X \rangle^m Q U H_u$  [19]. It is, however, not clear if the existence of such operators is consistent with the assumption that the operators  $K \sim X^\dagger X M^\dagger M$  ( $M = Q, U, D, L, E$ ) are absent.



|           | $SU(5)_{\text{hid}}$ | $SU(5)_{\text{SM}} \supset G_{\text{SM}}$ | $U(1)_H$  |
|-----------|----------------------|---|---|
| $X$       | $\mathbf{1}$         | $\mathbf{1}$                              | 2   |
| $F$       | $\square$            | $\overline{\square}$                      | -1  |
| $\bar{F}$ | $\overline{\square}$ | $\square$                                 | -1  |
| $H_u$     | $\mathbf{1}$         | $(\mathbf{1}, \mathbf{2})_{1/2}$          | 1   |
| $H_d$     | $\mathbf{1}$         | $(\mathbf{1}, \mathbf{2})_{-1/2}$         | 1   |
| $M$       | $\mathbf{1}$         | $3 \times (\overline{\square} + \square)$ | $-\frac{1}{2} + 3(x + \frac{1}{2})q_B + (y + \frac{1}{2})q_L$ |

Table 1: The entire matter content of the model. Here,  $H_u$ ,  $H_d$  and  $M$  represent the MSSM Higgs and matter fields, and  $q_B$  and  $q_L$  the baryon and lepton numbers, respectively.

## 7 Summary and Discussions

We have presented a simple and realistic model of supersymmetry breaking. The gauge group of the model is  $SU(5)_{\text{hid}} \times G_{\text{SM}}$ , and the matter content is given in Table 1. The form of the superpotential and Kähler potential interactions are controlled by the (anomalous) global  $U(1)_H$  symmetry, which can also be used to prohibit dangerous proton decay operators. With the higher dimension operators suppressed by the cutoff scale  $M_* \simeq 10^{16}$  GeV, the supersymmetry breaking masses and the  $\mu$  parameter are generated with the same order of magnitude. No flavor violating or  $CP$  violating terms arise.

The model requires the absence of direct interactions between the supersymmetry breaking and matter fields suppressed by the cutoff scale  $M_*$ . This should be understood as a property of the theory at or above  $M_*$ . The model also requires the coefficient  $\lambda$  of the superpotential interaction  $XF\bar{F}$  to satisfy  $\lambda^2 \lesssim 10^{-3}$ . (The coefficient  $\eta$  of the interaction  $F\bar{F}H_uH_d/M_*$  must also satisfy  $\eta \lesssim \lambda$ .) In particular, in order for our present analysis of the dynamics to be valid, the coefficient must be in the range  $10^{-6} \lesssim \lambda^2 \lesssim 10^{-3}$ . This range, however, is not that small. Moreover, since the bound arises from requiring the existence of a supersymmetry breaking minimum, a required value of  $\lambda$  may arise naturally as a result of anthropic selection.

The model provides several definite predictions on the spectrum of superparticles. The spectrum of the first two generation superparticles is that of minimal gauge mediation with the number of messengers  $N_{\text{mess}} = 5$  and the messenger scale  $M_{\text{mess}} \lesssim 10^{13}$  GeV. The condition  $M_{\text{mess}} \gtrsim 10^{11}$  GeV also arises if one focuses on the regime where the hidden sector gauge dynamics is perturbative at the messenger scale.<sup>11</sup> On the other hand, the spectrum of the Higgs bosons and third generation superparticles can have deviations from that of minimal gauge mediation

<sup>11</sup>A consistent supersymmetry breaking minimum exists for  $10^5$  GeV  $\lesssim M_{\text{mess}} \lesssim 10^{10}$  GeV in the case where the hidden sector gauge dynamics becomes strongly coupled above the messenger scale, as shown in Ref. [20].

because of the tree-level contributions to the Higgs mass-squared parameters,  $m_{H_u}^2$  and  $m_{H_d}^2$ , at  $M_*$ . The lightest supersymmetric particle is the gravitino with a mass of order  $(1 - 10)$  GeV.

How robust are these predictions under modifications to the model? There are several levels of modifications one can consider. For example, one can consider changing the hidden sector gauge group to  $SU(N_{\text{hid}})$  ( $N_{\text{hid}} > 5$ ), with  $F$  and  $\bar{F}$  transforming as  $\mathbf{5}^* + (N_{\text{hid}} - 1)\mathbf{1}$  and  $\mathbf{5} + (N_{\text{hid}} - 1)\mathbf{1}$  under  $SU(5)_{\text{SM}}$ . In this case, the prediction of  $N_{\text{mess}} = 5$  will be lost (it becomes  $N_{\text{mess}} > 5$ ), although the one for  $M_{\text{mess}}$  essentially remains (unless  $N_{\text{hid}}$  is much larger than 5). More drastic relaxations of the predictions, however, could also occur if we replace  $F$  and  $\bar{F}$  by  $F_{\text{hid}}(\mathbf{N}_{\text{hid}}, \mathbf{1}) + F_{\text{SM}}(\mathbf{1}, \mathbf{r}^*)$  and  $\bar{F}_{\text{hid}}(\mathbf{N}_{\text{hid}}^*, \mathbf{1}) + \bar{F}_{\text{SM}}(\mathbf{1}, \mathbf{r})$ , respectively, where the numbers in parentheses represent the transformation properties under  $SU(N_{\text{hid}}) \times SU(5)_{\text{SM}}$ . Here,  $N_{\text{hid}}$  is an arbitrary integer larger than 1, and  $\mathbf{r}$  is an arbitrary (in general reducible) representation of  $SU(5)_{\text{SM}}$ . In this case, the couplings for the  $XF_{\text{hid}}\bar{F}_{\text{hid}}$  and  $XF_{\text{SM}}\bar{F}_{\text{SM}}$  interactions can differ, and the only remaining prediction on  $N_{\text{mess}}$  and  $M_{\text{mess}}$  is  $M_{\text{mess}} \lesssim 10^{13}$  GeV, which is obtained by Eq. (20) with  $n_G^2 \rightarrow \dim(\mathbf{r})$ . The gravitino mass, however, is still of order  $(1 - 10)$  GeV.

As the LHC turns on, we will start acquiring the data on the masses and decay modes of the supersymmetric particles if weak scale supersymmetry is realized in nature. It will then be interesting to see if this data is consistent with (one of) the pattern(s) discussed in this paper. Such an exploration may shed some light on the origin of supersymmetry breaking, including the structure, e.g., the gauge group and matter content, of the hidden sector.

## Acknowledgment

This work was supported in part by the U.S. DOE under Contract DE-AC03-76SF00098, and in part by the NSF under grant PHY-04-57315. The work of Y.N. was also supported by the NSF under grant PHY-0555661, by a DOE OJI, and by an Alfred P. Sloan Research Foundation. M.P. thanks the Aspen Center for Physics, where a part of this work was performed.

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